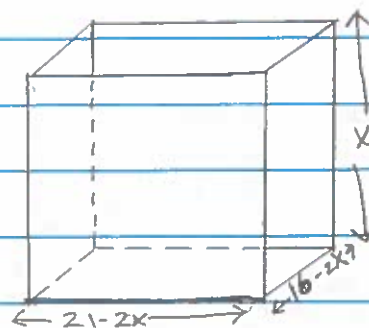
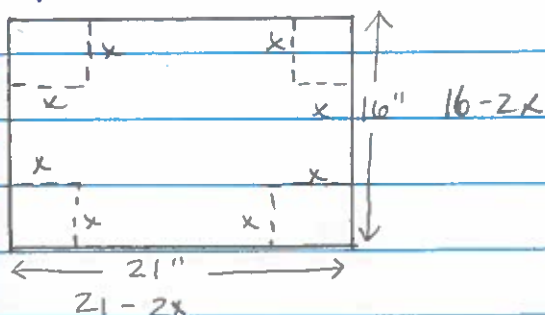


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$$0 \leq x \leq 8$$

$\therefore$ volume of box is

$$V = (21 - 2x)(16 - 2x)(x)$$

our box's value must be $V \geq 0$. We need

$$21 - 2x \geq 0 \quad \text{and} \quad 16 - 2x \geq 0 \quad \text{and} \quad x \geq 0$$

$$\Rightarrow x \leq \frac{21}{2} \quad \text{and} \quad x \leq \frac{16}{2} \quad \text{and} \quad x \geq 0$$

$$\Rightarrow x \leq 10.5 \quad \text{and} \quad x \leq 8 \quad \text{and} \quad x \geq 0$$

$$\therefore \text{Domain} = 0 \leq x \leq 8$$

$$V = (21 - 2x)(16 - 2x)(x); \quad 0 \leq x \leq 8$$

We need to find critical points

$$V = (4x^2 - 32x - 42x + 336)(x)$$

$$\Rightarrow V = 4x^3 - 74x^2 + 336x; \quad 0 \leq x \leq 8$$

$$\Rightarrow \frac{dV}{dx} = 12x^2 - 148x + 336$$

$$= 2(6x^2 - 74x + 168)$$

$$= 4(3x^2 - 37x + 84)$$

$$\text{When } \frac{dV}{dx} = 0, \quad 4(3x^2 - 37x + 84) = 0$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-37) \pm \sqrt{(-37)^2 - 4(3)(84)}}{2(3)}$$

$$= \frac{37 \pm \sqrt{361}}{6}$$

Ans. sp 2

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$$= 37 \pm \frac{19}{6}$$

$$\Rightarrow x = 37 + \frac{19}{6} \quad \text{or} \quad x = 37 - \frac{19}{6}$$

$$\Rightarrow x = \frac{56}{6} \quad \text{or} \quad x = \frac{18}{6}$$

$\Rightarrow x = 9\frac{1}{3}$ or $x = 3$ we reject $x = 9\frac{1}{3}$ as it is not on the domain, and $x = 3$ is accepted because it is.

When $\frac{dv}{dx} = \text{DNE}$, but $\frac{dv}{dx} = 12x^2 - 148x + 336$ exists every where on the domain,

$\therefore v = (21 - 2x)(16 - 2x)(x)$ has only one critical point ie $x = 3$.

We use the second derivative test.

$$\text{From } \frac{dv}{dx} = 12x^2 - 148x + 336$$

$$\Rightarrow \frac{dv}{dx} = 24(3) - 148$$

$$= 72 - 148$$

$$= -76 \leftarrow \text{negative}$$

Since $\frac{dv}{dx} = 0$ and $\frac{d^2v}{dx^2} = \text{negative}$ at $x = 3$, this critical point is a relative maximum point. According to the unique Extremum theorem, this is the only critical point on the domain of a continuous function, therefore this relative maximum point is also the absolute maximum point. The size of the corners that should be cut to produce a box with the maximum volume is a square of dimensions 3in. The max volume is

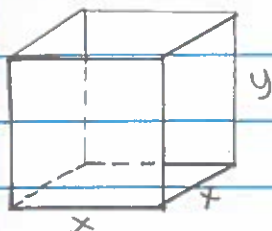
$$V_{\max} = (21 - 2(3))(16 - 2(3))(3) = (15)(10)(3) = 450 \text{ m}^3.$$

Ch 5.5 p3

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4p162



let x be the dimensions of the base

let y be the height of the box.

We want $V = 10 \text{ ft}^3$

$$\Rightarrow x^2 y = 10$$

cost of box, $C = (\text{cost of base}) + (\text{cost of sides}) + (\text{cost of top})$

$$= (3)(\text{surface of base}) + 2(\text{surface of sides}) + 2(\text{surf. of top})$$

$$\Rightarrow C = \$3(x^2) + \$2(4)(xy) + \$2(x^2)$$

$$\Rightarrow C = 3x^2 + 8xy + 2x^2$$

$$\Rightarrow C = 5x^2 + 8xy$$

From $x^2 y = 10 \Rightarrow y = \frac{10}{x^2} \leftarrow \text{sub into } C$

$$C = 5x^2 + 8x\left(\frac{10}{x^2}\right)$$

$$\Rightarrow C = 5x^2 + \frac{80}{x} \quad ; \quad x > 0$$

Now we find x that minimizes C . We need Critical points.

$$\frac{dC}{dx} = 10x + 80\left(-\frac{1}{x^2}\right) = \frac{10x^3}{x^2} - \frac{80}{x^2}$$

$$\Rightarrow \frac{dC}{dx} = \frac{10x^3 - 80}{x^2}$$

$$\text{When } \frac{dC}{dx} = 0, \Rightarrow \frac{10x^3 - 80}{x^2} = 0$$

$$\Rightarrow 10x^3 - 80 = 0 \Rightarrow x^3 = \frac{80}{10} = 8$$

$$\Rightarrow x = \sqrt[3]{8} = 2 \leftarrow \text{accepted as it is on the domain.}$$

$$\text{When } \frac{dC}{dx} = \text{DNE}, \Rightarrow \frac{10x^3 - 80}{x^2} = \text{DNE}$$

$$\Rightarrow x^2 = 0 \Rightarrow x = 0 \leftarrow \text{rejected as it is not on the domain.}$$

$$C = 5x^2 + \frac{80}{x} \text{ has only one critical point. i.e. } x = 2.$$

We need the second derivative test to classify this Critical point.

$$\text{From } \frac{dC}{dx} = 10x - \frac{80}{x^2} \Rightarrow \frac{d^2C}{dx^2} = 10 - 80\left(-\frac{2}{x^3}\right)$$

$$= 10 + \frac{160}{x^3} \quad \text{At the relative critical point } x = 2$$

$$\frac{d^2C}{dx^2} = 10 + \frac{160}{2^3} = \text{positive}$$

Ch 5.3 p4

m13:21

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∴ At this critical point, $\frac{dc}{dx} = 0$ and $\frac{d^2c}{dx^2} = \text{positive}$.

This critical point is a relative minimum point. According to the Unique Extremum theorem, this function is continuous on its domain and there is only one critical point on the domain, so therefore this relative minimum point is also the absolute minimum point. Therefore the cheapest cost for the box occurs when $x = 2\text{ft}$ and $y = \frac{10}{x^2} = \frac{10}{2^2} = \frac{10}{4} = 2.5\text{ft}$.

Economic Order Quantity.

Total production costs = storage costs for average inventory + Setup costs for each order

We want to minimize total production costs



If sales are consistent, the

$$\text{Average inventory} = \frac{q}{2}$$

where $q = \#$ of items per order

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let $n = \#$ of orders per year and $q = \#$ of items per order. Throwback Wear sells a total of 2400 pairs of jeans per year. $\Rightarrow nq = 2400$ ①

ordering costs, $C_o = \$4.50n$

Inventory costs, $C_I = 12\%$ of Value of avg inventory.

$$\Rightarrow C_I = (12\%)(\$8)\left(\frac{q}{2}\right) \leftarrow \text{since } \$8 \text{ price per pair}$$

$$\Rightarrow C_I = (0.12)(8)\left(\frac{q}{2}\right)$$

$$= 0.48q$$

Total costs, $C = C_o + C_I$

$$\Rightarrow C = 4.5n + 0.48q$$
 ②

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From ^① $nq = 2400$, $\Rightarrow n = \frac{2400}{q}$ $\leftarrow$ sub into ^②

$$C = 4.5 \left(\frac{2400}{q} \right) + 0.48q$$

$$\Rightarrow C = \frac{10800}{q} + 0.48q; \quad 1 \leq q \leq 2400$$

We want to minimize C . we need critical points starting with the first derivative.

$$\frac{dC}{dq} = 10800 \left(-q^{-1/2} \right) + 0.48$$

$$= -\frac{10800}{q^{1/2}} + 0.48$$

$$\Rightarrow \frac{dC}{dq} = \frac{0.48q - 10800}{q^{3/2}}$$

$$\text{When } \frac{dC}{dq} = 0, \Rightarrow \frac{0.48q^2 - 10800}{q^2} = 0.$$

$$\Rightarrow 0.48q^2 - 10800 = 0$$

$$\Rightarrow q^2 = \frac{10800}{0.48}$$

$$\Rightarrow q^2 = 22500 \Rightarrow q = \pm \sqrt{22500}$$

$\Rightarrow q = -150$ or $q = 150$. we reject $q = -150$ as it is not on the domain and accept $q = 150$ as it is.

$C = \frac{10800}{q} + 0.48q$ has only one critical point on its domain, i.e. $q = 150$.

We use the second derivative test to classify this

critical point, from $\frac{dC}{dq} = 0.48 - \frac{10800}{q^2}$,

$$\Rightarrow \frac{d^2C}{dq^2} = -10800 \left(\frac{-2q}{(q^2)^2} \right) = \frac{21600}{q^3}$$

When $q = 150$,

$$\Rightarrow \frac{d^2C}{dq^2} = \frac{21600}{150^3} = \text{Positive.}$$

At this critical point $q = 150$, $\frac{dC}{dq} = 0$ and $\frac{d^2C}{dq^2} = \text{positive}$.

So this point is a relative min. point. According to the unique Extremum theorem, This function is continuous on its domain and has only one critical point on the domain. Therefore, its relative minimum point is also its absolute minimum point.

Ch. 5.5p6

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The absolute minimum cost occurs when $q = 150$ and $n = \frac{2400}{q} = \frac{2400}{150} = 16$.

∴ Minimum costs occur when $n = 16$ orders made per year with each order consisting of $q = 150$ pairs of jeans. Minimum cost is

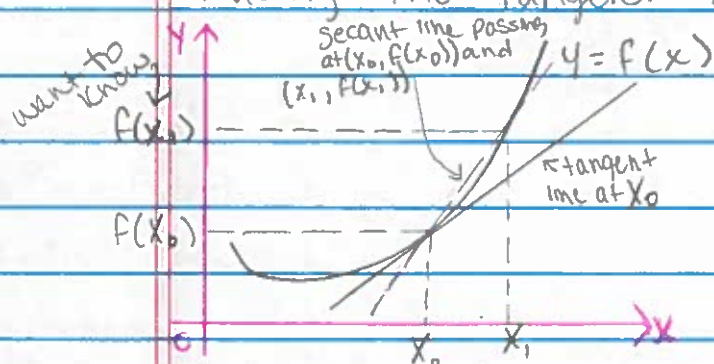
$$C = 4.5n + 0.48q$$

$$\Rightarrow 4.5(16) + 0.48(150) = \$144.$$

5.6

Differentials

→ Using the tangent line to approximate a function.



use the tangent line to approximate $y = f(x)$ at x_0 . We know the exact function value at x_0 , ie $f(x_0)$. We choose x_0 nearest to x_1 .

If these conditions are fulfilled, the slope of the secant line is approximated by the slope of the tangent line.

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx} \Rightarrow \Delta y \approx \frac{dy}{dx} \Delta x \Rightarrow \Delta y \approx f'(x_0) \Delta x$$

$$\text{We can expand this equation, } f(x_1) - f(x_0) \approx f'(x_0)(x_1 - x_0) \\ \Rightarrow f(x_1) \approx f(x_0) + f'(x_0)(x_1 - x_0)$$

Ex. Find $\sqrt{7}$.

let $f(x) = \sqrt{x}$ and $x_1 = 7$ since $f(x_1) = f(7) = \sqrt{7}$ ← want to find. Now we choose x_0 such that x_0 is close to $x_1 = 7$ and we know exactly $f(x_0) = \sqrt{x_0}$. Choose $x_0 = 9$ as it is closer than $x_0 = 4$ to 7. So, $f(x_0) = f(9) = \sqrt{9} = 3$.

$$\text{From } f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(x_0) = f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6} \\ \therefore f(x_1) \approx f(x_0) + f'(x_0)(x_1 - x_0) \Rightarrow \sqrt{7} \approx 2 \frac{2}{3}$$